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Statistical Λ −Convergence for Double Sequences in Probabilistic Normed Spaces

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ABSTRACT: Our main objective in this work is to describe new generalization of Λ-convergence for double sequences in the probabilistic normed spaces. The notion of statistical Λ-convergence and statistical Λ-Cauchy of double sequences has been defined in this particular normed space. We have given example which demonstrates that this idea is more generalized than the usual convergence.

Keywords: Statistical convergence, Statistical Λ-convergence, Probabilistic Normed Space (PN −space)

I. INTRODUCTION

Firstly, in 1951, statistical convergence was initially coined by Henry Fast [4]. In fact, Fast got this concept from Steinhaus [30]. Then Antoni Zygmund [32] was the first who proved statistical convergence for Fourier Series in his book “Trigonometric Series” [31] that was the first edition in 1935. But in that book, he was using the term ‘almost convergence’ rather than ‘statistical convergence’. The notion ‘almost convergence’ was already used by Lorentz [13] so Henry Fast [4] had to take some other name for his concept. So, he took ‘Statistical convergence’ as appropriate notion. After the papers of Fridy [5] and Šalãt [27], this concept became a dynamic area of research. After that, several authors contributed a vast literature for this concept. Several authors has given various extensions, generalizations, variants and applications about the notion [see 2, 6,12, 15,16, 18, 19, 22, 23, 24, 25, 28].

The term λ-statistical convergence for the sequences was introduced by Mursaleen [17]. He generalized this concept of statistical convergence using (V, λ)-summability. Further, the idea of statistical convergence for double sequences came into the consideration after the work of Bromwich [1]. Double sequences also were of keen interest of many researchers like Hardy [8], Tripathy [31], Mursaleen and Edely [20], Mursaleen and Mohiuddine [21], Kostyrko *et al.,* [3, 11] Savaş and Patterson [29] etc. in the area of “statistical convergence”.

The generalized metric space named as statistical metric space was presented by Menger [14]. Now days, it is known as probabilistic metric space and converted in active area of research. It has so many applications in functional analysis. These types of sequences has motivated Karakus [9, 10] to define a new concept statistical convergence for double sequence in PN-Space.

In this paper, we describe and analyze the term statistical Λ-convergence as well as statistical Λ- Cauchy for double sequences in the PN-Space. First, we review some basic terms as follows:

A natural density of the set E (which is the subset of natural numbers $\mathbb{N}$) is characterized by

$$\delta(E) = \frac{1}{n}|\{a \in E: a \leq n\}| \text{ , when } n \to \infty$$

where |.| indicates the order of the enclosed set.

Definition 1.1: [4] A sequence $x = (x_m)$ converges statistically to some number L if for each $\varepsilon > 0$,

$$\delta(\{m \leq n,: |x_m - L| \geq \varepsilon\}) = 0.$$

Symbollically, $St - \lim_{m\to\infty} x_m = L$ where St is the collection of all statistically convergent sequences.

Definition 1.2: The function $\varphi: R \to R$ is known as a distribution function when it is a non-decreasing and left continuous with (i) $inf_a \varphi(a) = 0$ and (ii) $sup_a \varphi(a) = 1$. $\mathcal{F}_+$ represents the collection of all the distribution functions.

Definition 1.3: A t-norm (triangular form) is a mapping $*:[0,1] \times [0,1] \to [0,1]$ which is continuous, non-decreasing, commutative and associative.

Definition 1.4: [7] Consider X is a linear space, $*$ is a t-norm and and $\mathcal{F}_+$ is the collection of distribution functions. Consider a map $P: X \to \mathcal{F}_+$ with $P_x = P(x)$ and $P_x(t)$ is the value of P_x at $t \in R$. Then P and $(X; P,*)$ is known as probabilistic norm and probabilistic normed space respectively, if it holds the next four axioms:

(i) $P_x(0) = 0,$
(ii) $P_x(t) = 1$, $\forall\, t > 0$ iff $x = 0$,
(iii) $P_{x\mu}(t) = P_x\left({t}/{|\mu|}\right)$ where $\mu \neq 0,$
(iv) $P_{x+y}(s+t) = P_x(s) * P_x(t)$, $\forall\, y, x \epsilon X$ and $s,t \in R_0^+ = [0,\infty)$.

Definition 1.5: [9] Let $(X, P,*)$ be a $PN - \text{Space}$. A double sequence $x = (x_{gh})$ is called convergent to some L with respect to P in $(X, P,*)$ if for any $\varepsilon > 0$ and $\varphi \in (0,1)$ $\exists$ a integer $N > 0$ such that $P_{x_{gh}-L}(\varepsilon) > 1 - \varphi$, whenever $g, h \geq N$.

Definition 1.6:[9] Let $(X, P,*)$ be a $PN - \text{Space}$. A double sequence $x = (x_{gh})$ is called Cauchy sequence with respect to P in $(X, P,*)$ if for any $\varepsilon > 0$ and $\varphi \in (0,1)$ $\exists\, N > 0$ and $M > 0$ such that $P_{x_{gh}-x_{pq}}(\varepsilon) > 1 - \varphi$, whenever $g, h \geq N$ and $p, q \geq M$.

II. MAIN RESULTS

Before explaining the statistical Λ-convergence for double sequences in $PN - \text{Space}$. First we mention Λ-convergence for single sequences which is defined by Mursaleen [26] as follows:

Suppose $\lambda = (\lambda_i)_{i=0}^{\infty}$ is a real sequence of positive numbers that approaches to ∞ with $\lambda_{n+1} \leq \lambda_n + 1, \lambda_1 = 1$. Then, sequence $x = (x_m)$ is termed as Λ-convergent to some L when $\Lambda_m(x) \to L$ whenever $m \to \infty$, and $\Lambda_m(x) = \frac{1}{\lambda_m}\sum_{i=0}^{m}(\lambda_i - \lambda_{i-1})\, x_i$, ($m\epsilon\mathbb{N}$).

Definition 2.1: A double sequence $x = (x_{gh})$ is called statistically Λ-convergent to some L if for any $\varepsilon > 0$,

$$\frac{1}{mn}\left|\{(g,h); g \leq n, h \leq m : \left|\Lambda x_{gh} - L\right| \geq \varepsilon\}\right| = 0 ,$$

when $m, n \to \infty$

i.e. $\delta_\Lambda(\{(g,h) \in \mathbb{N}\times\mathbb{N} : \left|\Lambda x_{gh} - L\right| \geq \varepsilon\}) = 0.$

We can denote it as $St_\Lambda - x_{gh} = L$.

Definition 2.2: A double sequence $x = (x_{gh})$ is called statistically Λ-Cauchy if for any $\varepsilon > 0$ $\exists\, N > 0$ and $M > 0$ such that

$\frac{1}{mn}\left|\{(g,h); g \leq n, h \leq m : \left|\Lambda x_{gh} - \Lambda x_{pq}\right| \geq \varepsilon\}\right| = 0$, when $m, n \to \infty$

i.e. $\delta_\Lambda(\{(g,h) \in \mathbb{N}\times\mathbb{N} : \left|\Lambda x_{gh} - \Lambda x_{pq}\right| \geq \varepsilon\}) = 0,$

whenever $g,\ p \geq N,\ h, q\ \geq M$.

Definition 2.3: Let $(X, P, *)$ be a $PN - \text{Space}$. A double sequence $x = (x_{gh})$ is called statistically Λ-convergent to some L with respect to P in $(X, P, *)$ if for any $\varepsilon > 0$ and $\varphi \in (0,1)$, we have

$\frac{1}{mn}\left|\left\{(g,h); g \leq n, h \leq m: P_{\Lambda x_{gh}-L}(\varepsilon) \leq 1-\varphi\right\}\right| = 0$, when $m, n \to \infty$.

i.e. $\delta_\Lambda\left(\left\{(g,h) \in \mathbb{N}\times\mathbb{N}: P_{\Lambda x_{gh}-L}(\varepsilon) \leq 1-\varphi\right\}\right) = 0.$

or

$\frac{1}{mn}\left|\left\{(g,h); g \leq n, h \leq m: P_{\Lambda x_{gh}-L}(\varepsilon) > 1-\varphi\right\}\right| = 1$,

when $m, n \to \infty$.

i.e. $\delta_\Lambda\left(\left\{(g,h) \in \mathbb{N}\times\mathbb{N}: P_{\Lambda x_{gh}-L}(\varepsilon) > 1-\varphi\right\}\right) = 1.$

Symbolically, $St_\Lambda^P - x_{gh} = L$.

Definition 2.4: Let $(X, P, *)$ be a $PN - \text{Space}$. A double sequence $x = (x_{gh})$ is called statistically Λ-Cauchy with respect to P in$(X, P, *)$ if for any $\varepsilon > 0$ and $\varphi \in (0,1)$ $\exists\, N > 0$ and $M > 0$ such that

$\delta_\Lambda\left(\left\{(g,h) \in \mathbb{N}\times\mathbb{N} :\ P_{\Lambda x_{gh}-\Lambda x_{pq}}(\varepsilon) \leq 1-\varphi\right\}\right) = 0,$

whenever $g, p \geq N\ , h, q \geq M$.

Theorem 2.5: A double sequence $x = (x_{gh})$ defined in $PN - \text{Space}(X, P, *)$ is statistically Λ-convergent with respect to P to unique limit.

Proof: Suppose that $St_\Lambda^P - x_{gh} = L_1$ and $St_\Lambda^P - x_{gh} = L_2$where $L_1 \neq L_2$.

For every $\varepsilon > 0$ and $\varphi \in (0,1)$, take $\theta \in (0,1)$ with $(1-\theta) * (1-\theta) \geq 1-\varphi$.

Let $K_1(\theta,\varepsilon) = \left\{(g,h) \in \mathbb{N}\times\mathbb{N}: P_{\Lambda x_{gh}-L_1}(\varepsilon) \leq 1-\theta\right\}$,

$K_2(\theta,\varepsilon) = \left\{(g,h) \in \mathbb{N}\times\mathbb{N}: P_{\Lambda x_{gh}-L_2}(\varepsilon) \leq 1-\theta\right\}$.

As $St_\Lambda^P - x_{gh} = L_1$, then we have $\delta_\Lambda\{K_1(\theta,\varepsilon)\} = 0$ for all $\varepsilon \geq 0$.

Furthermore, using $St_\Lambda^P - x_{gh} = L_2$, we have $\delta_\Lambda\{K_2(\theta,\varepsilon)\} = 0$ for all $\varepsilon \geq 0$.

Let $K_{1,2}(t,\varepsilon)$={$K_1(\theta,\varepsilon) \cup K_2(\theta,\varepsilon)$}, then we get $\delta_\Lambda\{K_{1,2}(\theta,\varepsilon)\} = 0$ i.e. $\delta_\Lambda\left(\mathbb{N}\times\mathbb{N} - K_{1,2}(\theta,\varepsilon)\right) = 1.$

If $(g,h) \in \mathbb{N}\times\mathbb{N} - K_{1,2}(\theta,\varepsilon)$, we have

$$\begin{aligned} P_{L_1-L_2}(\varepsilon) &\geq P_{\Lambda x_{gh}-L_1}\left(\tfrac{\varepsilon}{2}\right) * P_{\Lambda x_{gh}-L_2}\left(\tfrac{\varepsilon}{2}\right) \\ &> (1-\theta) * (1-\theta) \\ &\geq 1-\varphi. \end{aligned}$$

We get $P_{L_1-L_2}(\varepsilon)) = 1$as $\varphi > 0$ is arbitrary,

which gives $L_1 = L_2$.

Therefore, $St_\Lambda^P - x_{gh} = L$ is unique.

Theorem 2.6: If a double sequence $x = (x_{gh})$ is Λ-Convergent in $PN - \text{Space}(X, P, *)$ then it is also statistically Λ-convergent.

Proof: As sequence $x = (x_{gh})$is Λ-convergent in $(X, P, *)$ then for$\varepsilon > 0$ and $\varphi \in (0,1)\ \exists\, n_0 \in \mathbb{N}$ such that $P_{\Lambda x_{gh}-L}(\varepsilon) > 1-t$ for all $g \geq n_0$ and $h \geq n_0$.

Thus, $\delta_\Lambda\left\{(g,h) \in \mathbb{N}\times\mathbb{N}: P_{\Lambda x_{gh}-L}(\varepsilon) \leq 1-\varphi\right\} = 0.$

which establishes Theorem 2.6.

The next example can justify that the converse of above mentioned theorem may be not true.

Example 2.1: Consider $(R, |*|)$ is a real normed space with $P_x(t) = \frac{t}{t+|x|}$, $t \geq 0$ and $x \in R$.

Here $(R, P, |*|)$ is a $PN - \text{Space}$. Define sequence $x = (x_{gh})$ as

$$\Lambda x_{gh} = \begin{cases} \sqrt{gh} & , g \text{ and } h \text{ are squares} \\ 0 & \text{otherwise} \end{cases}$$

Now, for every $\varepsilon > 0$ and $\varphi \in (0,1)$,

$$\begin{aligned} \mathrm{N}_{m,n}(\varphi,\varepsilon) &= \left\{(g,h); g \leq n, h \leq m: P_{\Lambda x_{gh}}(\varepsilon) \leq 1-\varphi\right\} \\ &= \left\{(g,h); g \leq n, h \leq m: \frac{t}{t+\left|\Lambda x_{gh}\right|} \leq 1-\varphi\right\} \\ &= \left\{(g,h); g \leq n, h \leq m : \left|\Lambda x_{gh}\right| \geq \frac{\varphi t}{1-\varphi} > 0\right\} \\ &= \left\{(g,h); g \leq n, h \leq m : \left|\Lambda x_{gh}\right| = \sqrt{gh}\right\} \\ &= \left\{(g,h); g \leq n, h \leq m :\ g \text{ and } h \text{ are squares}\right\} \end{aligned}$$

Then,

$$\frac{1}{nm}\left|\mathrm{N}_{m,n}(\varphi,\varepsilon)\right| \leq \frac{1}{nm}\left|\{(g,h); g \leq n, h \leq m\right. \\ \left. : g \text{ and } h \text{ are squares}\}\right| \leq \frac{\sqrt{nm}}{nm} = 0.$$

$$\Rightarrow St_\Lambda^P - \lim\, x = 0.$$

Thus, above sequence is statistically Λ -convergent but not usually convergent in $(R, P, |*|)$.

Theorem 2.7: Let $x = (x_{gh})$ and $y = (y_{gh})$ be two sequences in $PN - \text{Space}\ (X, P, *)$. Then

(i) If $St_\Lambda^P - x_{gh} = x_0$ and $St_\Lambda^P - y_{gh} = y_0$, then $St_\Lambda^P - (x_{gh} + y_{gh}) = x_0 + y_0$.

(ii) If $St_\Lambda^P - x_{gh} = x_0$and $St_\Lambda^P - cx_{gh} = cx_0$where $c \in R$.

Proof : (i) Let $St_\Lambda^P - x_{gh} = x_0$ and $St_\Lambda^P - y_{gh} = y_0$.

For $\varepsilon > 0$ and $\varphi \in (0,1)$, take $\theta \in (0,1)$ with $(1-\theta) * (1-\theta) \geq 1-\varphi$.

Let $K_1(\theta,\varepsilon) = \left\{(g,h) \in \mathbb{N}\times\mathbb{N} : P_{\Lambda x_{gh}-l}(\varepsilon) \leq 1-\theta\right\}$,

$K_2(\theta,\varepsilon) = \left\{(g,h) \in \mathbb{N}\times\mathbb{N} : P_{\Lambda y_{gh}-l}(\varepsilon) \leq 1-\theta\right\}$.

Since, $St_\Lambda^P - x_{gh} = x_0$, we have $\delta_\Lambda\{K_1(\theta,\varepsilon)\} = 0$ for all $\varepsilon > 0$.

Similarly, since $St_\Lambda^P - y_{gh} = y_0$, we get $\delta_\Lambda\{K_2(\theta,\varepsilon)\} = 0$ for all $\varepsilon > 0$

Take $K(\theta,\varepsilon)$={$K_1(\theta,\varepsilon) \cup K_2(\theta,\varepsilon)$}.

Then we observe that $\delta_\Lambda\{\mathrm{K}(\theta,\varepsilon)\} = 0$ which implies $\delta_\Lambda\{K^c(\theta,\varepsilon)\} = 1$.

If $(g,h) \in \mathbb{N}\times\mathbb{N} - K(\theta,\varepsilon)$, then we have

$$\begin{aligned} P_{(\Lambda x_{gh}-x_0)+(\Lambda y_{gh}-y_0)}(\varepsilon) &\geq P_{\Lambda x_{gh}-x_0\left(\frac{\varepsilon}{2}\right)} * P_{\Lambda y_{gh}-y_0\left(\frac{\varepsilon}{2}\right)} \\ &> (1-\theta) * (1-\theta) \\ &\geq 1-\varphi. \end{aligned}$$

This shows that

$$\delta_\Lambda\left(\left\{(g,h) \in \mathbb{N}\times\mathbb{N}: P_{(\Lambda x_{gh}-x_0)+(\Lambda y_{gh}-y_0)}(\varepsilon) \leq 1-\theta\right\}\right) = 0.$$

So $St_\Lambda^P - (x_{gh} + y_{gh}) = x_0 + y_0$.

(ii) Let $St_\Lambda^P - x_{gh} = x_0$.

First we take $c = 0$. For $\varepsilon > 0$ and $\varphi \in (0,1)$

$$P_{\Lambda 0x_{gh}-0x_0}(\varepsilon) = P_0(\varepsilon) = 1 > 1 - \varphi$$
$$P_{\Lambda 0x_{gh}} - 0x_{gh} = 0.$$

Then by Theorem 2.6, we have $St_\Lambda^P - \lim 0x_{gh} = 0$.

Now, take $c \in R$ but $(c \neq 0)$.

Since $St_\Lambda^P - 0x_{gh} = x_0$, then for every $\varepsilon > 0$ and $\varphi \in (0,1)$, we define the set

$A(\varphi,\varepsilon) = \left\{(g,h) \in \mathbb{N}\times\mathbb{N}: P_{\Lambda x_{gh}-x_0}(\varepsilon) \leq 1-\varphi\right\}$ such that $\delta_\Lambda\{\mathrm{A}(\varphi,\varepsilon)\} = 0$.

In this case $\delta_\Lambda\{A^c(\varphi,\varepsilon)\} = 1$.

If $(g,h) \in \{A^c(\varphi,\varepsilon)\}$, then

$$P_{\Lambda cx_{gh}-cx_0}(\varepsilon) = P_{\Lambda x_{gh}-x_0}\left(\frac{\varepsilon}{|c|}\right)$$
$$\geq P_{\Lambda x_{gh}-x_0}(\varepsilon) * P_0\left(\frac{\varepsilon}{|c|} - \varepsilon\right)$$
$$= P_{\Lambda x_{gh}-x_0}(\varepsilon) * 1$$
$$> 1 - t.$$

For $c \in R (c \neq 0)$ this shows that

$\delta_\Lambda\left(\left\{(g,h) \in \mathbb{N}\times\mathbb{N}: P_{\Lambda cx_{gh}-cx_0}(\varepsilon) \leq 1-t\right\}\right) = 0.$

Hence $St_\Lambda^P - cx_{gh} = cx_0$.

Theorem 2.8: A double sequence $x = (x_{gh})$ in $PN-\mathrm{Space}(X,P,*)$ is statistically Λ-convergent to L iff there exists a set $H = \{(g,h)\} \subseteq \mathbb{N}\times\mathbb{N}; g,h = 1,2,3\ldots$ with $\delta_\Lambda(H) = 1$ and $St_\Lambda^P - x_{gh} = L$.

Proof: Firstly we assume that $St_\Lambda^P - x_{gh} = L$, then for every $\varepsilon > 0$ and $d \in \mathbb{N}$.

Take $K(d,\varepsilon) = \left\{(g,h) \in \mathbb{N}\times\mathbb{N} : P_{\Lambda x_{gh}-L}(\varepsilon) \leq 1 - \frac{1}{d}\right\}$,

$$M(d,\varepsilon) = \left\{(g,h) \in \mathbb{N}\times\mathbb{N} : P_{\Lambda x_{gh}-L}(\varepsilon) > 1 - \frac{1}{d}\right\}.$$

Then $\delta\{K(d,\varepsilon)\} = 0$ with

$M(1,\varepsilon) \supset M(2,\varepsilon) \ldots \supset M(i,\varepsilon) \supset M(i+1,\varepsilon) \ldots.$ (a)

and $\delta_\Lambda\{M(d,\varepsilon)\} = 1,\ d = 1,2,\ldots$ (b)

Next, we contrary prove the required result. Assume sequence $x = (x_{gh})$ is not statistically Λ-convergent to L. Then, for any $\varepsilon > 0$ and $\varphi \in (0,1)$, we have has infinitely many terms in the set $\{(g,h) \in \mathbb{N}\times\mathbb{N} : P_{\Lambda x_{gh}-L}(\varepsilon) \leq 1-\varphi\}$.

Take $M(\varphi,\varepsilon) = \left\{(g,h) \in \mathbb{N}\times\mathbb{N}: P_{\Lambda x_{gh}-L}(\varepsilon) > 1 - \varphi\right\}; \varphi > \frac{1}{d} (d = 1,2,\ldots)$

Then $\delta_\Lambda(M(\varphi,\varepsilon)\} = 0$.

$\therefore$ by (a) $M(d,\varepsilon) \subset M(\varphi,\varepsilon)$. Hence $\delta_\Lambda\{M(d,\varepsilon)\} = 0$ which is contradiction to (b).

Therefore $x = (x_{gh})$ is statistically $\Lambda -$ convergent to L.

Conversely,

Let there exists a set $H = \{(g,h): g,h = 1,2,\ldots\} \subset \mathbb{N}\times\mathbb{N}$ with $\delta_\Lambda(H) = 1$ and $St_\Lambda^P - x_{gh} = L$. Then, for any $\varepsilon > 0$ and $\varphi \in (0,1)\ \exists\ N \in \mathbb{N}$ with

$$P_{\Lambda x_{gh}-L}(\varepsilon) > 1 - \varphi, \forall\ g,h \geq N.$$

Now,

$$M(\varphi,\varepsilon) = \left\{(g,h) \in \mathbb{N}\times\mathbb{N}: P_{\Lambda x_{gh}-L}(\varepsilon) > 1 - \varphi\right\}$$
$$\subseteq (\mathbb{N}\times\mathbb{N}) - \{(g_{k_0+1},h_{k_0+1}),(g_{k_0+2},h_{k_0+2}),\ldots\}$$

Therefore, $\delta_\Lambda\{M(\varphi,\varepsilon)\} \leq 1 - 1 = 0.$

Hence, $St_\Lambda^P - x_{gh} = L$.

Theorem 2.9: A double sequence $x = (x_{gh})$ is statistically Λ-convergent in $PN - \mathrm{Space}(X,P,*)$ iff it is statistically Λ-Cauchy in $PN - \mathrm{Space}(X,P,*)$.

Proof: Let $St_\Lambda^P - x_{gh} = L$.

For $\varepsilon > 0$ and $\varphi \in (0,1)$, we have

$\delta_\Lambda\left(\left\{(g,h) \in \mathbb{N}\times\mathbb{N}: P_{\Lambda x_{gh}-L}(\varepsilon) \leq 1-\varphi\right\}\right) = 0.$

Take two numbers M and N such that

$$A(\varphi,\varepsilon) = \left\{(g,h) \in \mathbb{N}\times\mathbb{N}: P_{\Lambda x_{gh}-\Lambda x_{MN}}(\varepsilon) \leq 1-\varphi\right\},$$
$$B(\varphi,\varepsilon) = \left\{(g,h) \in \mathbb{N}\times\mathbb{N}: P_{\Lambda x_{gh}-\mathrm{L}}(\varepsilon) \leq 1-\varphi\right\},$$
$$C(\varphi,\varepsilon) = \left\{(g,h) \in \mathbb{N}\times\mathbb{N}: P_{\Lambda x_{MN-\mathrm{L}}}(\varepsilon) \leq 1-\varphi\right\}.$$

Then $A(\varphi,\varepsilon) \subseteq B(\varphi,\varepsilon) \cup C(\varphi,\varepsilon)$

i.e. $\delta_\Lambda\{A(\varphi,\varepsilon)\} \leq \delta_\Lambda\{B(\varphi,\varepsilon)\} + \delta_\Lambda\{C(\varphi,\varepsilon)\} = 0.$

Thus $x = (x_{gh})$ is statistically Λ-Cauchy.

Conversely,

Assume $x = (x_{gh})$ is statistically Λ-Cauchy, then $\delta_\Lambda\{A(\varphi,\varepsilon)\} = 0$.

We contrary prove that sequence statistically Λ-convergent. Assume sequence $x = (x_{gh})$ is not statistically Λ-convergent then $\delta_\Lambda\{B(\varphi,\varepsilon)\} = 1$ i.e. $\delta_\Lambda\{(\mathbb{N}\times\mathbb{N}) - B(\varphi,\varepsilon)\} = 0$. We can write

$$P_{\Lambda x_{gh}-\Lambda x_{MN}}(\varepsilon) \leq 2P_{\Lambda x_{gh}-\mathrm{L}}(\varepsilon/2) < 1-\varphi$$

if $P_{\Lambda x_{gh}-\mathrm{L}}(\varepsilon/2) < \frac{1-\varphi}{2}$.

As $\delta_\Lambda\{(B(\varphi,\varepsilon)^c\} = 0 \Rightarrow \delta_\Lambda\{(A(\varphi,\varepsilon)^c\} = 0$ i.e $\delta_\Lambda\{A(\varphi,\varepsilon)\} = 1$ which is a contradiction.

III. CONCLUSIONS

This paper presents the overview of statistical convergence in setup of PN –spaces by defining and studying the idea of statistical Λ-convergence as well as statistical Λ-Cauchy for double sequences. Here, we derived more generalized results than the analogous results for double sequences in PN-spaces.

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CONFLICT OF INTEREST

Authors have no any conflict of interest.

REFERENCES

[1]. Bromwich, T. J. I. (1965). An Introduction to the Theory of Infinite Series. *MacMillan & Co., New York*.

[2]. Çolak, R. and Bektas, C. A. (2011). λ-statistical convergence of order α. *Acta Mathematica Scientia*, **31B**(3): 953–959.

[3]. Das, P., Kostyrko, P., Wilczyncki, W. & Malik, P. (2008). I and I∗-convergence of double sequence. *Math. Slovaca*, **58**(5): 605–620.

[4]. Fast, H. (1951). Sur la convergence statistique. *Colloq. Math*., **2**: 241–244.

[5].Fridy, J. A. (1985). On statistical convergence. *Analysis*, **5**(4): 301–313.

[6]. Fridy, J. A. & Orhan, C. (1993). Lacunary statistical convergent. *Pacific J. Math*., **160**(1): 43-51.

[7]. Gӓhler, S. (1963). 2-metrische Raume und ihretopologeische Struktur, *Math. Nachr.,* **26**,115-148.

[8]. Hardy, G. H. (1917). On the convergence of certain multiple series. *Proc. Camb. Phil. Soc*., **19**: 86-95.

[9]. Karakus, S. (2007). Statistical convergence on probabilistic normed spaces. *Math. Commun*., **12**: 11-23.

[10]. Karakus, S. & Demirci, K. (2007). Statistical convergence of double sequences on probabilistic

normed spaces. *Inter J. Math. Math. Sci*., doi:10.1155/2007/14737

[11]. Kostyrko, T., Salat & Wilczynski, W. (2000/2001). I-convergence. *Real Anal. Exchange*, **26**(2): 669–686.

[12]. Kumar, V. & Mursaleen, M. (2011). On λ, μ-statistical convergence of double sequences on intuitionistic fuzzy normed spaces. *Filomat*, **25**(2): 109–120.

[13]. Lorentz, G. G. (1948). A contribution to the theory of divergent sequences. *Acta Mathematica*, **80**: 167–190.

[14]. Menger, K. (1942). Statistical metrics. *Proc. Nat. Acad. Sci. USA*, **28**: 535-537.

[15]. Mohiuddine, S. A. & Aiyub, M. (2012). Lacunary statistical convergence in random 2-normed spaces. *Appl. Math. Info. Sci*., **6**(3): 581–585.

[16]. Mohiuddine, S. A. & Lohani, Q.M.D. (2009). On generalized statistical convergence in intuitionistic fuzzy normed space. *Chaos Solitons Fractals*, **42**(3):, 1731–1737.

[17]. Mursaleen, M. (2000). λ-statistical convergence. *Math. Slovaca,* **50**: 111–115.

[18]. Mursaleen, M. & Alotaibi, A. (2011). On I-convergence in random 2-normed spaces. *Math. Slovaca*, **61**(6): 933–940.

[19]. Mohuiddine, S. A., Alotaibi, A. & Alsulami, S. M. (2012). Ideal convergence of double sequences in 2 random 2-normed spaces. *Advances in Difference Equations*, Vol. 2012, article 149.

[20]. Mursaleen, M. & Edely, O. H. H. (2003). Statistical convergence of double sequences. *J. Math. Anal. Appl*., **288**: 223–231.

[21]. Mursaleen, M. & Mohiuddine, S. A. (2009). Statistical convergence of double sequences in intuitionistic fuzzy normed spaces. *Chaos Solitons Fractals*, **41**: 2414–2421.

[22]. Mursaleen, M. & Mohiuddine, S. A. (2009). On lacunary statistical convergence with respect to the intuitionistic fuzzy normed space. *J. Comput. Appl. Math*., **233**(2): 142–149.

[23]. Mursaleen, M. & Mohiuddine, S. A. (2010). On ideal convergence of double sequences inprobabilistic normed spaces. *Mathematical Reports*, **12**(4): 359–371.

[24]. Mursaleen, M. & Mohiuddine, S. A. (2012). On ideal convergence in probabilistic normed spaces. *Math. Slovaca*, **62**(1): 49–62.

[25]. Mursaleen, M., Mohiuddine, S. A. & Edely, O.H.H. (2010). On the ideal convergence of double sequences in intuitionistic fuzzy normed spaces. *Computers & Mathematics with Applications*, **59**(2): 603–611.

[26]. Mursaleen, M. & Noman, A. K. (2010). On the spaces of λ-convergent and bounded sequences. *Thai J. Math*, **8**(2): 311–329.

[27]. Šalãt, T. (1980). On statistically convergent sequences of real numbers. *Math. Slovaca*, **30**: 139–150.

[28]. Savaş¸ E. & Mohiuddine, S. A. (2012). λ-statistically convergent double sequences in probabilistic normed spaces. *Math. Slovaca*, **62**(1): 99–108.

[29]. Savaş, E. and Patterson, R.F. (2006). Lacunary statistical convergence of multiple sequences. *Applied Mathematical Letters*, **19**: 527-534.

[30]. Steinhaus, H. (1951). Sur la convergence ordinaireet la convergence asymptotique. *Colloq. Math*., **2**: 73–34.

[31]. Tripathy, B. C. (2003). Statistically convergent double sequences. *Tamkang J. Math*., **34**(3): 231-237.

[32]. Zygmund, A. (1959). Trigonometric Series. *Cambridge Univ. Press, New York, NY, USA*, 1st edition.